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We point out that the gravitational-wave event GW150914 observed by the LIGO detectors can be explained by the coalescence of primordial black holes (PBHs). It is found that the expected PBH merger rate would exceed the rate estimated by the LIGO Scientific Collaboration and the Virgo Collaboration if PBHs were the dominant component of dark matter, while it can be made compatible if PBHs constitute a fraction of dark matter. Intriguingly, the abundance of PBHs required to explain the suggested lower bound on the event rate, > 2 events $\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$, roughly coincides with the existing upper limit set by the nondetection of the cosmic microwave background spectral distortion. This implies that the proposed PBH scenario may be tested in the not-too-distant future.

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Introduction.—The gravitational-wave event GW150914 observed by the LIGO detectors [1] revealed the existence of black holes (BHs) with a mass of around $30M_\odot$ in the form of binaries. Although there are several possible explanations for the origin of those BHs as well as the formation of the binaries (see Ref. [2] and references therein), the answer is yet to be elucidated. Assuming all the BH binaries relevant to the LIGO observation have the same physical parameters, such as masses and spins, as those of GW150914, the merger event rate was estimated as 2–53 $\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$ [3].

In this Letter, we discuss the possibility that the event GW150914 was caused by a merger of a primordial BH (PBH) binary. PBHs are BHs that have existed since the very early epoch in cosmic history before any other astrophysical object had been formed [4]. The most popular mechanism to produce PBHs is the direct gravitational collapse of a primordial density inhomogeneity [5,6]. If the primordial Universe were highly inhomogeneous [$O(1)$ in terms of the comoving curvature perturbation] on superhorizon scales, as realized in some inflation models (see Ref. [7] and references therein), an inhomogeneous region upon horizon reentry would undergo gravitational collapse and form a BH. The mass of the BH is approximately equal to the horizon mass at the time of formation, $M_{\mathrm{BH}} \sim 30\,M_\odot[(4\times 10^{11})/(1+z_f)]^2$, where z_f is the formation redshift. Thus, it is possible that PBHs with a mass of around $30M_\odot$ are formed deep in the radiation-dominated era.

The event rate of the PBH binary mergers has been already given in Ref. [8] for the case where PBHs are massive compact halo objects with their mass around $0.5M_\odot$ and constitute the dominant component of dark matter. In Ref. [8] it was found that two neighboring PBHs having a sufficiently small separation can form a binary in the early Universe and coalesce within the age of the Universe. We apply the formation scenario in Ref. [8] to the present case where the PBHs are about $30M_\odot$ and the fraction of PBHs in dark matter is a free parameter. We present a detailed computation of the event rate in the next section. The resultant event rate turns out to exceed the event rate mentioned above (2–53 $\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$) if PBHs are the dominant component of dark matter. Intriguingly, however, it falls in the LIGO range if PBHs are a subdominant component of dark matter with the fraction that nearly saturates the upper limit set by the nondetection of the cosmic microwave background (CMB) spectral distortion due to gas accretion onto PBHs [9].

Recently, it was claimed in Ref. [10] (see also Ref. [11]) that the event GW150914 as well as the event rate estimated by LIGO can be explained by the merger of PBHs even if PBHs are the dominant component of dark matter. Our study differs from Ref. [10] in the following two points: (1) the formation process of PBH binaries and (2) the fraction of PBHs in dark matter. First, in Ref. [10] PBH binaries are assumed to be formed due to energy loss by gravitational radiation when two PBHs accidentally pass by each other with a sufficiently small impact parameter. This mechanism is different from what we consider in this Letter (see the next section). Second, in Ref. [10] the fraction of PBHs in dark matter to explain the estimated gravitational-wave event rate by the LIGO-Virgo Collaboration is of order unity, while in our case we require it to be as small as the upper limit obtained in Ref. [9]. Namely, our claim is that PBHs as a small fraction of dark matter can explain the event rate suggested by the detection of GW150914.

Throughout this Letter, we set the speed of light to be unity, $c = 1$.

Event rate of mergers of PBH binaries.—In this section, we estimate the event rate of PBH binary mergers. We adopt the formation mechanism proposed in Ref. [8] and basically follow the same calculation procedure described in it. A refined analysis taking into account various effects neglected in Ref. [8] shows that those effects can change the event rate estimation at most by ~50% [12]. Given the large uncertainties in the event rate estimated by the LIGO-Virgo Collaboration as well as in the upper limit on the abundance of PBHs from the nondetection of the CMB spectral distortion, those corrections are not important and we adopt the simple method given in Ref. [8]. Our analysis differs from that in Ref. [8] in two aspects: the PBH mass is $30M_\odot$, and the PBH fraction in dark matter is a free parameter. For simplicity, we assume all the PBHs have the same mass. If necessary, our analysis can be straightforwardly generalized to a realistic situation in which the PBH mass function is not monochromatic.

Let f be the fraction of PBHs in dark matter, i.e., $\Omega_{\rm BH} = f\Omega_{\rm DM}$. Then, the physical mean separation $\bar{x}$ of BHs at matter-radiation equality at the redshift $z = z_{\rm eq}$ is given by

$$\bar{x} = \left(\frac{M_{\rm BH}}{\rho_{\rm BH}(z_{\rm eq})}\right)^{1/3} = \frac{1}{(1+z_{\rm eq})f^{1/3}}\left(\frac{8\pi G}{3H_0^2}\frac{M_{\rm BH}}{\Omega_{\rm DM}}\right)^{1/3}. \quad (1)$$

Let us consider two neighboring BHs separated by a physical distance x at matter-radiation equality. The pair decouples from the expansion of the Universe and becomes gravitationally bound when the average energy density of the BHs over the volume R^3, where R is the separation of two BHs, exceeds the background cosmic energy density ρ, that is, when

$$M_{\rm BH}R^{-3} > \rho(z). \quad (2)$$

Using $R = [(1+z_{\rm eq})/(1+z)]x$, the redshift at which the decoupling occurs is given by

$$\frac{1+z_{\rm dec}}{1+z_{\rm eq}} = f\left(\frac{\bar{x}}{x}\right)^3 - 1 > 0. \quad (3)$$

This shows that only a pair having $x < f^{1/3}\bar{x}$ can form a binary. From now on, we require this condition on x. If there are only two BHs on top of the unperturbed Friedmann-Lemaitre-Robertson-Walker Universe, after being decoupled from the background expansion, they move closer together and finally collide without forming a binary. In a realistic situation, other BHs are also present and the third BH closest to the BH pair affects the infall motion of the BHs in the pair by giving them the a tidal force. As a result, the head-on collision does not happen and the BHs in the pair form a binary typically having a large eccentricity. The major and minor axes of the binary at the formation time are given by (denoted by a and b, respectively)

$$a = \frac{\alpha}{f}\frac{x^4}{\bar{x}^3}, \qquad b = \beta\left(\frac{x}{y}\right)^3 a, \quad (4)$$

where y is the physical distance to the third BH at $z = z_{\rm eq}$ and α and β are numerical factors of $O(1)$. A detailed investigation of the dynamics of the binary formation suggests $\alpha = 0.4$, $\beta = 0.8$ [12]. In the following analysis, we take $\alpha = \beta = 1$ for simplicity. In Fig. 1 showing our estimated merger event rate, the event rate in the case $\alpha = 0.4$, $\beta = 0.8$ is also plotted, which demonstrates that the difference between the two is not significant compared to the uncertainty of the event rate provided by the LIGO-Virgo Collaboration.

The eccentricity of the binary at the formation time is given by

$$e = \sqrt{1 - \left(\frac{x}{y}\right)^6}. \quad (5)$$

By definition, $y > x$ must be satisfied. In addition to this, we also have the condition $y < \bar{x}$, which yields an upper bound on e as

$$e_{\rm max} = \sqrt{1 - f^{3/2}\left(\frac{a}{\bar{x}}\right)^{3/2}}. \quad (6)$$

We assume a uniform probability distribution both for x and y in three-dimensional space [13]. Thus, the probability dP that the would-be binary BHs have a separation in $(x, x+dx)$ and that the distance to the perturber BH is in $(y, y+dy)$ is given by

$$dP = \frac{9}{\bar{x}^6}x^2y^2dxdy. \quad (7)$$

We can convert this probability distribution function into the one for a and e by using the mapping formulas (4) and (5). The result is given by

$$dP = \frac{3}{4}f^{3/2}\bar{x}^{-3/2}a^{1/2}e(1-e^2)^{-3/2}da\,de. \quad (8)$$

Once the BHs form a binary, they gradually shrink by gravitational radiation and eventually merge. The coalescence time is given by [14,15]

$$t = Qa^4(1-e^2)^{7/2}, \qquad Q = \frac{3}{170}(GM_{\rm BH})^{-3}. \quad (9)$$

Using this equation, we can convert the probability distribution above into the one defined in the $t-e$ plane as

$$dP = \frac{3}{16}\left(\frac{t}{T}\right)^{3/8} e(1-e^2)^{-(45/16)}\frac{dt}{t}de,$$
$$T \equiv \frac{\bar{x}^4 Q}{f^4}. \quad (10)$$

Integrating this probability density over e for fixed t, we obtain the probability distribution function for the coalescing time. The upper limit of e is given by

$$e_{\rm upper} = \begin{cases} \sqrt{1-\left(\frac{t}{T}\right)^{6/37}}, & \text{for } t < t_c, \\ \sqrt{1-f^2\left(\frac{t}{t_c}\right)^{2/7}}, & \text{for } t \geq t_c, \end{cases} \tag{11}$$

where t_c is defined by

$$t_c = Q\bar{x}^4 f^{25/3}. \tag{12}$$

The probability that the coalescence occurs in the time interval $(t, t+dt)$ then becomes

$$dP_t = \begin{cases} \frac{3}{58}\left[-\left(\frac{t}{T}\right)^{3/8}+\left(\frac{t}{T}\right)^{3/37}\right]\frac{dt}{t}, & \text{for } t<t_c, \\ \frac{3}{58}\left(\frac{t}{T}\right)^{3/8}\left[-1+\left(\frac{t}{t_c}\right)^{-(29/56)}f^{-(29/8)}\right]\frac{dt}{t}, & \text{for } t\geq t_c. \end{cases} \tag{13}$$

The probability that the coalescence happens within the time interval (0, t) is then simply given by $P_c(t) = \int_0^t dP_t$. The LIGO-Virgo Collaboration obtained the event rate 2–53 $\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$ for the BH binary coalescence from the observation of the event GW150914 at $z = z_{\rm GW150914} = 0.09$ [3]. It is not a trivial task to compare the event rate of PBH coalescence with that given by the LIGO-Virgo Collaboration in a rigorous manner since the event rate is assumed to be uniform in comoving volume and source time in Ref. [3] while this is not true in our case. Here, we simply ignore the effects of cosmological evolution and consider the event rate evaluated at the present time, which is obtained by taking the limit $\lim_{\Delta t\to 0}\{[P_c(t_0) - P_c(t_0-\Delta t)]/\Delta t\}$, where t_0 is the age of the Universe. Thanks to the relatively low value of $z_{\rm GW150914}$, this approximation is valid within the accuracy we are care about. Indeed, changing $\Delta t = 0$ to Δt corresponding to the average redshift of the observed volume, $\bar{z} \approx 0.15$, shifts the event rate only by less than 25%.

The present average number density of PBHs $n_{\rm BH}$ is given by

$$n_{\rm BH} = \frac{3H_0^2}{8\pi G}\frac{\Omega_{\rm BH}}{M_{\rm BH}}. \tag{14}$$

Then, the event rate becomes

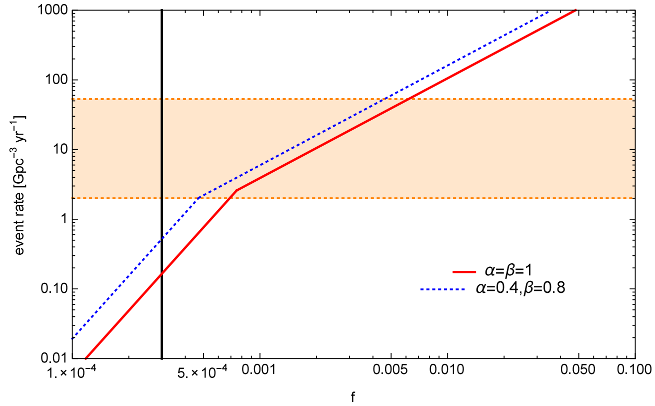

FIG. 1. Event rate of mergers of $30M_\odot$-$30M_\odot$ PBH binaries as a function of the PBH fraction in dark matter $f = \Omega_{\rm BH}/\Omega_{\rm DM}$. The red line is the case for $\alpha = \beta = 1$, which we have employed throughout the calculations. The blue dotted line is the case for $\alpha = 0.4$, $\beta = 0.8$ suggested in Ref. [12]. The event rate estimated by the LIGO-Virgo Collaboration is shown as the shaded region colored orange. The black solid line at $f \approx 3\times 10^{-4}$ is the upper limit on f from the nondetection of the CMB spectral distortion obtained in Ref. [9].

$$\text{event rate} = n_{\rm BH}\lim_{\Delta t\to 0}\frac{P_c(t_0)-P_c(t_0-\Delta t)}{\Delta t} = \frac{3H_0^2}{8\pi G}\frac{\Omega_{\rm BH}}{M_{\rm BH}}\frac{dP_c}{dt}\bigg|_{t_0}. \tag{15}$$

Figure 1 shows the event rate (15) as a function of f. We adopt $\Omega_{\rm DM} = 0.27$ and $H_0 = 70$ km/Mpc s. We find that the event rate falls in the LIGO range if the fraction f is around 10^{-3}. As a comparison, we also show the case with $\alpha = 0.4$, $\beta = 0.8$ [see Eq. (4)]. Given that our event rate is derived from the order of magnitude argument, the difference between the two cases is reasonably acceptable. Furthermore, the difference of the event rates for any f is smaller than the uncertainty of the event rate from the LIGO-Virgo Collaboration, which also justifies our simplified analysis.

Discussion.—It is quite intriguing that the event rate of mergers of PBH binaries falls into the range of that given by the LIGO-Virgo Collaboration when the fraction of PBHs in dark matter nearly saturates the upper limit obtained by the nondetection of the CMB spectral distortion [9]. In other words, our PBH scenario may be experimentally falsifiable in the not-too-distant future. It should be noted that the upper limit hinges on the various approximations made to deal with the complexity of the accretion process and it is not an easy task to quantify how uncertain the result of Ref. [9] is. Because of this, we consider the upper limit as the order of magnitude estimation. Furthermore, it has been assumed that PBHs initially distribute uniformly in space. Depending on the statistical properties of the primordial perturbations, this assumption is not necessarily satisfied and it is possible that PBHs upon formation are clustered. In the latter case, it is expected that the binary formation becomes more efficient and the merger event rate is enhanced compared to the present case. Another potentially important effect that we did not take into account is the dynamical friction acting on the BHs in the binaries caused by the interaction with dark matter trapped in the gravitational potential of the binaries. If the PBH fraction f is as small as the value corresponding to the kink in Fig. 1, the mass of the trapped dark matter becomes comparable to the BH mass at the matter-radiation equality and grows further in the matter dominated era. Since the time scale of the dynamical friction is much shorter than the age of the Universe, it may be possible that the binary size quickly changes by a factor of $O(1)$. Quantifying this effect on the event rate is beyond the scope of this Letter (see Ref. [16] for the related discussion). With the coincidence between our estimated event rate and the observation within the uncertainties mentioned above, we conclude that the event GW150914 could be a PBH binary merger.

Let us briefly mention that it is unlikely that the PBH binary is disrupted by other compact objects such as other PBHs and stars. The typical major axis of the PBH binary for a given lifetime of the binary, which we take to be the age of the Universe t_0, is given as a solution of $t_0 = Qa^4(1-e_{\rm max}^2)^{7/2}$ since the possible largest eccentricity is the most probabilistically favored. We then find that $a \approx 7\times 10^4$ A.U.$(f/f_c)^{-28/37}$ for $f \geq f_c$ and $a \approx 7\times 10^4$ A.U. for $f \leq f_c$, where $f_c \approx 7\times 10^{-4}$ is f at the kink in Fig. 1. Since the probability that a given PBH binary is disrupted by the compact objects becomes smaller for smaller f if $f < f_c$, we now focus on $f \geq f_c$. The PBH binary will be disrupted if the velocity gain of the PBH due to the gravitational force by the incident compact object becomes comparable to the orbital velocity of the binary. Denoting by d the closest distance that the compact object approaches the PBH in the binary, the velocity gain is roughly estimated as $Gm/(vd)$, where v is the typical relative velocity between the binary and the compact object and m is the mass of the compact object. Then, the maximum d for which disruption occurs is written as $d_{\rm max} \approx a(m/M_{\rm BH})(v_{\rm orbital}/v)$, where $v_{\rm orbital}$ is the velocity of the PBHs in the binary. Thus, the probability that a given PBH binary collides with other compact objects within the age of the Universe is estimated as $P\sim d_{\rm max}^2 n v t_0$, where n is the number of the compact objects considered. Using this formula, for the PBH binary residing in dark matter halos like the Milky Way, we have

$$P \sim 7\times 10^{-8}\left(\frac{v}{200\ {\rm km/s}}\right)^{-1}\left(\frac{M_{\rm halo}}{10^{12}M_\odot}\right)\left(\frac{L_{\rm halo}}{100\ {\rm kpc}}\right)^{-3} \times \left(\frac{f}{f_c}\right)^{9/37}\left(\frac{m}{30\,M_\odot}\right)^2, \qquad (16)$$

where $M_{\rm halo}$ and $L_{\rm halo}$ are the halo mass and halo size, respectively. Thus, such PBH binaries are not likely to be disrupted. If the PBH binary resides in a stellar environment like a galactic disk, the disruption probability is

$$P \sim 3\times 10^{-3}\left(\frac{v}{200\ {\rm km/s}}\right)^{-1}\left(\frac{f}{f_c}\right)^{-(28/37)} \times \left(\frac{n_{\rm star}}{1\ {\rm pc}^{-3}}\right)\left(\frac{m}{1M_\odot}\right)^2. \qquad (17)$$

Thus, such PBH binaries are also likely to survive for the age of the Universe. From these estimations, we conclude that most PBH binaries are not disrupted by encounters with other compact objects.

At present, we do not know how to discriminate the PBH scenario from other astrophysical scenarios (see, e.g., Refs. [2,17]). For instance, a scenario based on Population III binaries also explains the high event rate with the peak of the BH mass distribution around $\sim 30M_\odot$ [18,19] (see also Ref. [20]). However, as we mentioned above, the approximate coincidence of the estimated fraction of PBHs in dark matter with the upper limit from the absence of CMB spectral distortions implies that our PBH scenario may be experimentally proved in the near future. In particular, the proposed experiment PIXIE [21] is supposed to measure the CMB spectral distortion down to the level of 10^{-8}, which is roughly a 4 orders of magnitude improvement from the current sensitivity. Since the CMB spectral distortion produced by the PBHs is proportional to the fraction f, the upper limit on f will be also improved by 4 orders of magnitude if no spectral distortion is detected. Thus, if the PBH scenario proposed in this Letter turns out to be the case, future CMB experiments should detect the spectral distortion with great significance, which is a distinctive feature of this scenario. On the other hand, if that were not to happen, the PBH scenario would be strongly disfavored.

Another potentially useful method to discriminate the PBH scenario from the others is to exploit the distribution of two BH masses in binaries. A number of detections of BH mergers, which will occur true in the coming years, will bring us information about the distribution of the binary parameters. It was demonstrated in Ref. [17] that the binary distribution may be used to reconstruct the initial mass function of BHs in an astrophysical scenario where the mergers occur in dense stellar systems. It is worth studying the binary distribution in the PBH scenario and it will be interesting to see if a distinct feature specific to this scenario appears.

Last, but not least, another interesting direction to pursue is to investigate the low frequency stochastic gravitational-wave background continuously emitted by binary PBHs that are still in the phase of orbital motion [22]. We naturally expect the presence of an enormous number of such PBH binaries in the present Universe. It would be interesting to determine the spectrum of such gravitational waves and clarify whether it helps to test our PBH scenario.

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